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Research on Position Control of Robotic Arm Based on Inversion Method

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Abstract: The research on robotic arm position control is not only a key component of the development of robot technology, but also an important force to promote the process of industrial automation and intelligence. The basic principle of the inversion control method is to transform the control problem of a nonlinear system into a control problem of a linear system by constructing the virtual control input of the system, thereby simplifying the design process of the controller. On this basis, a position control strategy based on inversion method was proposed for the mathematical model of the manipulator. The strategy fully considers the dynamic characteristics and nonlinear factors of the robotic arm, designs the corresponding controller, and proves the stability of the closed-loop system. The Simulnk platform in MATLAB software is used to design the controller based on the inversion method and simulate the analysis, and the results show that the robotic arm based on the inversion controller has a good improvement in tracking accuracy, response speed and anti-interference ability.

Keywords: Robotic Arm; Flexible Joints; Dynamics; Position Control; Inversion Method

1 INTRODUCTION

In today's wave of industrial automation and intelligence, the position control accuracy and flexibility of the robotic arm, as a key equipment to perform complex tasks, have become an important indicator to measure its performance. The movement of the robotic arm mainly depends on the rotation of its joints and the expansion and contraction of the connecting rod, through which the position and attitude of the end effector in space are changed[1-2]. The key to making the robot arm move is the design of the control system, in which the commonly used control algorithm includes PID control[2-6], adaptive control[1-9], fuzzy control[4-11], neural network control[12], Sliding Mode Control ） [13-16], Backstepping[12-17]. Although traditional robotic arm position control methods, such as PID control and fuzzy control, have achieved good control results within a certain range, their control performance is often difficult to meet the needs of high-precision operations in the face of strong nonlinearity, model uncertainty and external interference.

As a control method for nonlinear systems, the backstepping method can effectively deal with the uncertainty and nonlinear factors in the system and improve the stability and accuracy of the position control of the manipulator by constructing the Lyapunov function step by step. Through the hierarchical control method, the inversion method simplifies the design process of the controller, making the control tasks of each level clearer and easier to implement. The inversion method can consider the uncertainty and external interference of the system, improve the robustness of the control system, and make it maintain good performance under various working conditions. Although the inversion method is based on model design and requires real-time calculation of the inverse model, with the improvement of computing power, it can meet the requirements of real-time control.

The purpose of this study is to explore the position control strategy of the robotic arm based on the inversion method, and to achieve accurate tracking and stable control of the position of the robotic arm by constructing a suitable inversion controller. First, we will build a dynamic model of the robotic arm[18]and analyze its nonlinear properties. Then, the inversion method is used to design the position controller, and the stability of the control system is proved by the Lyapunov stability theory. Finally, the effectiveness and superiority of the proposed method are verified by simulation experiments and practical applications.

2 DESIGN OF THE INVERSION CONTROLLER OF THE ROBOTIC ARM

2.1 FUNDAMENTALS OF THE INVERSION

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METHOD

Firstly, the high-order complex cascaded nonlinear system is decomposed into several subsystems, and the number of subsystems does not exceed the order of the system. Secondly, the intermediate virtual control quantity and Lyapunov function (referred to as V function) are designed for the first subsystem to ensure its stability, and then the intermediate virtual control quantity and Lyapunov function are designed for the upper level system, which is retreated to the whole system, and finally the whole system is integrated to complete the design of the whole inversion control, so as to realize the comprehensive adjustment and tracking of the whole system, so that the complex system can achieve the expected performance indicators. The inversion design method is suitable for indeterminate nonlinear systems that can be linearized or have strict parameter feedback.

2.2 INVERT THE CONTROLLER DESIGN

Take the dynamic equation of the expression of the flexible joint manipulator, which can be expressed as follows:

$$\begin{cases} I\ddot{q}_1 + Mgl\sin q_1 + K(q_1 - q_2) = 0 \\ J\ddot{q}_2 + K(q_2 - q_1) = u \end{cases} \tag{1}$$

where indicates the I moment of inertia of the joint; M Indicates the quality of the connecting rod; g denotes gravitational acceleration; l Indicates the center of gravity of the connecting rod to the length of the joint; u Indicates motor torque input; q_1 Indicates the angle of the joint; q_2 Indicates the rotation angle of the motor; The parameter K represents the stiffness of the flexible joint. K The larger the value, the greater the elastic stiffness of the joint, the smaller the joint flexibility, and q_1 q_2 the closer it is to the joint. K The lower the value, the reverse is true.

Let ,$x_1 = q_1$ $x_2 = q_2$ then the expression (2-1) of the flexible joint manipulator can be expressed as:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{1}{I}(Mg\sin x_1 + K(x_1 - x_3)) \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \dfrac{1}{J}(u - K(x_3 - x_1)) \end{cases} \tag{2}$$

In view of the above dynamic equations, the controller is designed based on the inversion method to solve the problem of inaccurate position control of the flexible joints of the manipulator, and the design steps are as follows:

Step 1

First, define the first position error signal:

$$z_1 = x_1 - x_{1d} \tag{3}$$

where x1d is the desired trajectory of the joint. Define the first virtual control quantity $a_1 = -c_1 z_1$, where c_1 is a constant.

Define the second position error signal as

$$z_2 = x_2 - a_1 - \dot{x}_{1d} \tag{4}$$

z_1 Derivatives are obtained:

$$\dot{z}_1 = \dot{x}_1 - \dot{x}_{1d} = x_2 - \dot{x}_{1d} = z_2 + a_1 \tag{5}$$

Define the first Lyapunov function as $V_1 = \frac{1}{2}z_1^2$, and derive its time to get :

$$\dot{V}_1 = z_1(z_2 + a_1) = -cz_1^2 + z_1 z_2 \tag{6}$$

When $z_2 = 0$,$\dot{V}_1 \leq 0$; However z_2, it does not have to be equal to 0, so a new amount of virtual control needs to be introduced to make it 0.

Step two

Define the second Lyapunov function as $V_2 = V_1 + \frac{1}{2}z_2^2$, and derive its time to yield:

$$\dot{V}_2 = -c_1 z_1^2 + z_1 z_2 + z_2 \dot{z}_2 \tag{7}$$

z_2 Derivation:

$$\begin{aligned} \dot{z}_2 &= \dot{x}_2 + c_1 \dot{z}_1 - \ddot{x}_{1d} \\ &= -\frac{1}{I}(Mgl\sin x_1 + K(x_1 - x_3)) + c_1(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d} \end{aligned} \tag{8}$$

Substituting Eq. (2-8) into Eq. (2-7) yields:

$$\begin{aligned} \dot{V}_2 &= -c_1 z_1^2 + z_1 z_2 + z_2\left[-\frac{1}{I}(Mgl\sin x_1 + K(x_1 - x_3)) + c_1(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d}\right] \\ &= -c_1 z_1^2 + z_1 z_2 + z_2\left[-\frac{1}{I}(Mgl\sin x_1 + Kx_1) + c_1(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d}\right] + \frac{K}{I}z_2 x_3 \end{aligned} \tag{9}$$

Define the third position error signal as

$$z_3 = x_3 - a_2 \tag{10}$$

Among them a_2 is the second virtual control quantity.

take

$$a_2 = -\frac{I}{K}\left[-\frac{1}{I}(Mgl\sin x_1 + Kx_1) + c_1(x_2 - \dot{x}_{1d}) - \ddot{x}_{1d} + z_1 + c_2 z_2\right] + z_3 \tag{11}$$

among others $c_2 > 0$.

First substitute Eq. (2-11) into Eq. (2-10), and then substituting Eq. (2-10) into Eq. (2-9) to obtain

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 + \frac{K}{I} z_2 z_3 \tag{12}$$

When $z_3 = 0$, $\dot{V}_2 \leqslant 0$. However z_3, it is not necessarily equal to 0, so it is necessary to continue to introduce new virtual control quantities to make z_3 it equal to 0. Step 3

Define the third Lyapunov function as $V_3 = V_2 + \frac{1}{2} z_3^2$, and derive its time as:

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 + \frac{K}{I} z_2 z_3 + z_3 \dot{z}_3 \tag{13}$$

z_3 Derivation:

$$\dot{z}_3 = x_4 + \frac{I}{K}\left[-\frac{1}{I}(Mgl\cos x_1 x_2 + Kx_2) + c_1(\dot{x}_2 - \ddot{x}_{1d}) - \dddot{x}_{1d} + \dot{z}_1 + c_2 \dot{z}_2\right] \tag{14}$$

Let S, take as:

$$S = -\frac{1}{I}(Mgl\cos x_1 x_2 + Kx_2) + c_1(\dot{x}_2 - \ddot{x}_{1d}) - \dddot{x}_{1d} + \dot{z}_1 + c_2 \dot{z}_2 \tag{15}$$

Substituting Eq. (2-14) and Eq. (2-15) into Eq. (2-13) yields:

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 + \frac{K}{I} z_2 z_3 + z_3\left(x_4 + \frac{I}{K} S\right) \tag{16}$$

Define the fourth position error signal:

$$z_4 = x_4 - a_3 \tag{17}$$

take $a_3 = -\left(c_3 z_3 + \frac{K}{I} z_2 + \frac{I}{K} S\right)$, where, $c_3 > 0$. Then there are:

$$\dot{V}_3 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 + z_3 z_4 \tag{18}$$

When $z_4 = 0$, $\dot{V}_3 \leqslant 0$. However z_4, it is not necessarily 0, so it is necessary to further introduce a virtual control quantity to make z_4 it equal to 0.

Step 4:

Respectively $\dot{z}_1$, $\dot{z}_2$ seek derivation, get

$$\ddot{z}_1 = -\frac{1}{I}(Mgl\sin x_1 + K(x_1 - x_3)) - \ddot{x}_{1d} \tag{19}$$

$$\ddot{z}_2 = -\frac{1}{I}(Mglx_2\cos x_1 + K(x_2 - x_4)) - c_1\left(\left(-\frac{1}{I}(Mgl\sin x_1 + K(x_1 - x_3))\right) - \ddot{x}_{1d}\right) - \dddot{x}_{1d} \tag{20}$$

Right, S to get it

$$\dot{S} = -\frac{1}{I}(-Mgl\sin x_1 \cdot x_2^2 + K\dot{x}_2) + c_1(\ddot{x}_2 - \dddot{x}_{1d}) - \ddddot{x}_{1d} + \ddot{z}_1 + c_2 \ddot{z}_2 \tag{21}$$

To z_4 the derivative, got

$$\dot{z}_4 = \frac{u}{J} - \frac{K}{J}(x_3 - x_1) + \frac{I}{K}\dot{S} + c_3 \dot{z}_3 + \frac{K}{I}\dot{z}_2 \tag{22}$$

Define the fourth Lyapunov function as $V_4 = V_3 + \frac{1}{2} z_4^2$, and take the derivative of its time, which yields

$$\dot{V}_4 = \dot{V}_3 + z_4\left(\frac{u}{J} - \frac{K}{J}(x_3 - x_1) + \frac{I}{K}\dot{S} + c_3 \dot{z}_3 + \frac{K}{I}\dot{z}_2\right) \tag{23}$$

In order to make $\dot{V}_4 \leqslant 0$ it, the final design control law is

$$u = -J\left(-\frac{\dot{K}}{J}(x_3 - x_1) + \frac{I}{K}\dot{S} + c_3 \dot{z}_3 + \frac{K}{I}\dot{z}_2 + z_3 + c_4 z_4\right) \tag{24}$$

among others $c_4 > 0$. The derivative of the fourth Lyapunov function at this point

$$\dot{V}_4 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 - c_4 z_4^2 \leqslant 0 \tag{25}$$

That is $\dot{V}_4 \leqslant 0$, it is proved that the controller designed based on the inversion method can make the system meet the theoretical conditions of Lyapunov's stability, and the four position errors are asymptotically stable, so as to ensure the exponential asymptotic stability of the system in a global sense, and z_1 the

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exponential form asymptotically converges to 0, which proves that the designed control law is theoretically correct.

3 SIMULATIONS AND RESULTS

The values of each parameter in the simulation process are $I=1.0kg\cdot m^2$, $J=1.0kg\cdot m^2$, , $Mgl=5N\cdot m$, $K=500N\cdot m$, Let the expected trajectory of the joint be and $x_{1d}=\sin(t)$ the controller parameter is . $c_1=c_2=c_3=c_4=50$

The simulation program was built based on the Simulnk platform, and the simulation results are shown below.

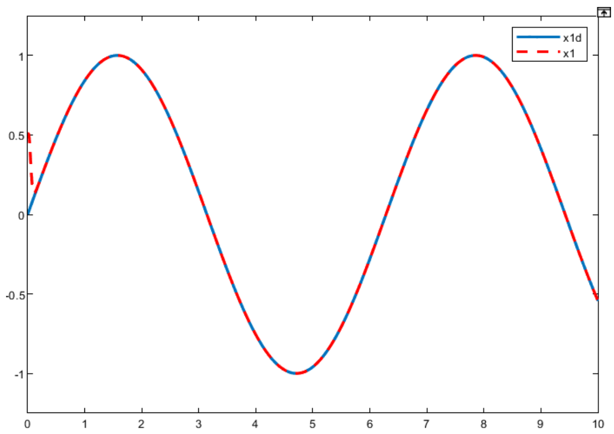

FIGURE 1 ANGULAR TRAJECTORY TRACKING OF CONNECTING RODS

In Figure 3-1, the x-axis coordinates represent the time, the y-axis coordinates represent the linkage angle, x1 represents the actual linkage angle, and x1d represents the desired trajectory.

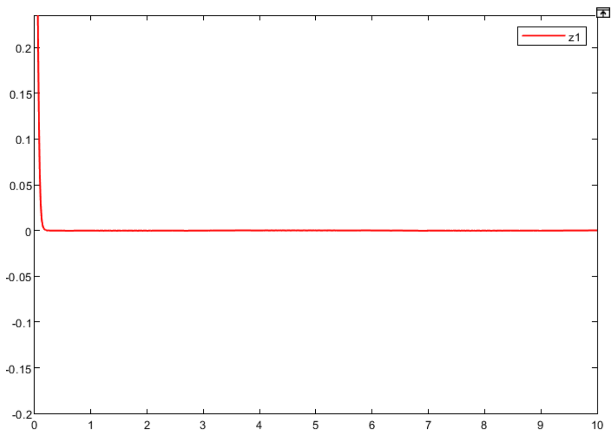

FIGURE 2 LINKAGE ANGLE TRACKING ERROR

The x-axis coordinates in Figure 3-2 represent the time, the y-axis coordinates represent the tracking error, and Z1 represents the linkage angular tracking error.

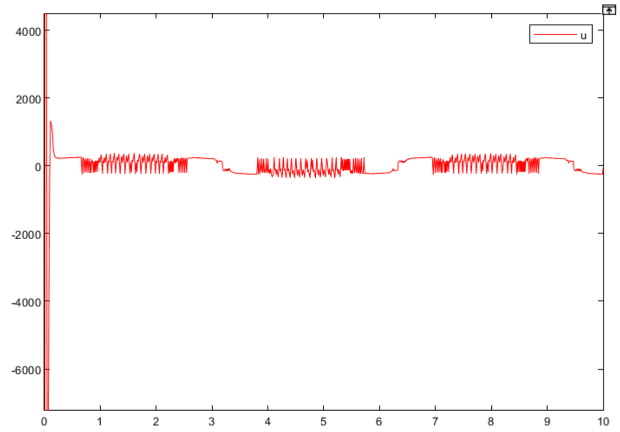

FIGURE 3 TORQUE CONTROL

In Figure 3-3, the x-axis coordinates represent the time, the y-axis coordinates represent the magnitude of the control torque, and the u coordinates represent the control torque.

As can be seen from Fig. 3-1 and Fig. 3-2, the actual trajectory of the joint quickly tracked the expected trajectory within 1 second after the simulation began, and there was no situation that it did not follow for a certain period of time, which can preliminarily show that the designed inversion controller has achieved the expected effect.

4 CONCLUSIONS

In this research on controlling the position of the robotic arm based on the inversion method, we deeply explore the application of the inversion method in the field of robotic arm position control. Through rigorous theoretical analysis and a large number of experimental verifications, we have successfully constructed a robotic arm position control model based on the inversion method. The model makes full use of the unique advantages of the inversion method to decompose the complex manipulator system into multiple subsystems for processing, which effectively solves the limitations of traditional control methods in dealing with multivariate, strong coupling and nonlinearity.

In the research process, we designed an adaptive inversion controller for the uncertainties existing in the robotic arm system, such as the perturbation of model parameters, external interference and unmodeled dynamics, by using the inversion method combined with adaptive control technology. This controller can estimate the uncertainty of the system in real time, and adjust the control law online according to the estimation results, so that the robotic arm can still maintain high-precision position control performance in complex and changeable working environments. Through simulation experiments and actual hardware experiments, we compare the performance of the control strategy based on the inversion method with the traditional PID control and sliding mode control methods. The experimental results clearly show that the control strategy based on the inversion method shows significant advantages in position tracking accuracy, anti-interference ability and system stability, which effectively improves the reliability and accuracy of the manipulator under complex working conditions.

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REFERENCES

[1]Hu S ,Wan Y ,Liang X . Adaptive nonsingular fast terminal sliding mode trajectory tracking control for robotic manipulators with model feedforward compensation[J]. Nonlinear Dynamics,2025,(prepublish):1-19.

[2]ZHANG Pingshe,HE Zhiwei. Research on motion control of multi-degree-of-freedom manipulator[J].Science and Technology Innovation and Productivity,2025,46(03):100-102.)

[3]Zhang Y ,He L ,Chen J , et al. Neural Network Adaptive Hierarchical Sliding Mode Control for the Trajectory Tracking of a Tendon-Driven Manipulator[J]. Chinese Journal of Mechanical Engineering,2025,38(1):18-18.

[4]Wang H ,Jiao X ,Wu S , et al. Model-Less Tracking Control for Continuum Manipulators Based on Fuzzy Adaptive Zeroing Neural Networks[J]. International Journal of Fuzzy Systems,2025,(prepublish):1-19.

[5]Zhang W . Predefined-Time Fuzzy Control for Robotic Manipulators Driven by Compliant Actuators[J]. International Journal of Fuzzy Systems,2025,(prepublish):1-10.

[6]Armenta M M ,Avelar A C ,Gandarilla I , et al. Solving trajectory tracking of robot manipulators via PID control with neural network compensation[J]. Soft Computing,2025,29(2):1-15.

[7]Wang J ,Cui Y . Adaptive neural network tracking control for robotic manipulator with input dead zone and function constraints on states[J]. Nonlinear Dynamics,2025,(prepublish):1-19.

[8]Yang Y ,Su Q ,Hui W , et al. Adaptive model-free finite-time tracking control for flexible-link manipulator with parametric and non-parametric uncertainties[J]. Nonlinear Dynamics,2025,(prepublish):1-17.

[9]Zhao K ,Xie Y ,Xu S , et al. Adaptive neural appointed-time prescribed performance control for the manipulator system via barrier Lyapunov function[J]. Journal of the Franklin Institute,2025,362(2):107468-107468.

[10]LI Xiyuan, LIN Chunqing, CHEN Hailei, et al. Abnormal vibration control at the end of manipulator arm based on PID parameter optimization[J].Machinery Manufacturing and Automation,2024,53(06):251-256.DOI:10.19344/j.cnki.issn1671-5276.2024.06.050.

[11]Jiaqing F ,Lei Z ,Dongyu T . Integrated sliding mode control of robot manipulator based on fuzzy adaptive RBF[J]. Xibei Gongye Daxue Xuebao/Journal of Northwestern Polytechnical University,2024,42(6):1099-1110.

[12]PANG Aimin,WANG Zhen,MA Shuangbao. Adaptive neural network inversion control of flexible manipulator[J].Mechanical Design and Manufacturing,2024,(06):309-314.DOI:10.19356/j.cnki.1001-3997.2024.06.008.

[13]Sheikh M S ,Jafar M S ,S.M. S H , et al. Sliding-surface dynamic control of a continuum manipulator with large workspace[J]. Control Engineering Practice,2023,141

[14]Tao X ,Youqun Z ,Fen L , et al. Backstepping sliding-mode control for active suspension system matching mechanical elastic wheel with uncertainties[J]. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering,2022,236(12):2696-2710.

[15]Yuhang L ,Lei Z ,Jiaxin Z , et al. Backstepping Adaptive Trajectory Tracking Control of Manipulator with Uncertainties of Model and State[J]. Journal of Physics: Conference Series,2022,2224(1):

[16]Guiying L ,Shuyang W ,Zhigang Y . Adaptive nonlinear observer - based sliding mode control of robotic manipulator for handling an unknown payload[J]. Proceedings of the Institution of Mechanical Engineers,2021,235(3):302-312.

[17]Hongyu C ,Xiucheng D ,Yong Y , et al. Backstepping Sliding Mode Control of Uncertainty Flexible Joint Manipulator with Actuator Saturation[J]. Journal of Physics: Conference Series,2021,1828(1):012165-.

[18]HUANG Hua. Research on modeling and control of flexible joint manipulator[D].Hunan University of Technology,2014.