



EHL Oil Film Thickness Analysis of VH-CATT Cylindrical Gear

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Abstract: Variable hyperbolic circular arc tooth trace (VH-CATT) cylindrical gear can be used in high-speed and heavy-duty occasions because of its good transmission performance and bearing capacity. In order to analyze the influence of different working conditions on the oil film thickness, the tooth surface contact mathematical model of VH-CATT cylindrical gear is constructed, and the tooth surface contact point and main curvature of VH-CATT cylindrical gear system are calculated. Based on EHL theory, the minimum oil film thickness and central oil film thickness model of VH-CATT cylindrical gear EHL are constructed, and the distribution of oil film thickness on the tooth surface of VH-CATT cylindrical gear and the variation law of minimum oil film thickness and central oil film thickness in one meshing cycle are analyzed. The results show that as the input speed increases, the entrainment speed increases, and the corresponding oil film thickness increases significantly. When the input torque becomes larger, the decreasing trend of oil film thickness is much smaller than the amplitude of torque change. This study provides a theoretical basis for predicting gear life and calculating wear.

Keywords: VH-CATT cylindrical gear ; contact analysis ; bullet oil lubrication ; oil film thickness

1 INTRODUCTION

Variable hyperbolic circular arc tooth trace (VH-CATT) cylindrical gear is a new type of gear. Because its tooth trace is named circular arc, it is formed by double-edged milling of large cutter head. Compared with traditional gear, this gear has more stable transmission performance, higher meshing rate and larger bearing capacity[1].

The contact form of VH-CATT cylindrical gear is point contact, which becomes elliptical contact after loading, and has good oil film forming conditions on the tooth surface. Based on the above advantages, VH-CATT cylindrical gears have a very wide range of application prospects, and many scholars have carried out research. For example, Yan Lili et al[2]. provided a basic theoretical tool for gear contact stress analysis based on Hertz classical contact theory. Ma Dengqiu[3] combined Hertz theory and fractal theory to build a contact mechanics model, which refined the analysis dimension of gear contact force. Based on the mathematical equation of tooth surface and the basic theory of gear meshing, Wei Yongqiao et al[4] systematically explored the core geometric characteristics of VH-CATT cylindrical gear transmission, and sorted out the structure and transmission correlation law of the new gear in detail. Ma[5] established the hypothesis of gear meshing contact impact, and established the mechanical model of gear meshing contact impact. Zhang[6] obtained the normal contact force and contact pressure of the tooth surface according to the finite element simulation method.

In summary, the previous research on the gear focused on the contact characteristics, meshing stiffness, etc., and lacked the calculation of the oil film thickness.

When studying the oil film thickness and oil film pressure distribution of gear transmission, the model and equation modeling are introduced into Reynolds[7] equation. The oil film pressure and film thickness values solved are different from the real measured values under the actual working conditions of the gear, such as ignoring the key factors such as tooth surface curvature change, tooth surface extrusion effect and elastic deformation, lubricant temperature rise and viscosity attenuation[8], but this is a pioneering attempt for the hydrodynamic lubrication mechanism of gear transmission fluid, which lays a foundation for the follow-up study of EHL thickness. Grubin[9] obtained the EHL theory of line contact after considering the above influencing factors, and summarized the formula of oil film thickness in the center of line contact.

Subsequently, the oil film thickness was studied by Dowson[10] and the minimum and center oil film thickness formulas were proposed. Zhou[11] built an isothermal elastohydrodynamic lubrication model of the gear transmission, and disassembled the dynamic change mechanism of oil film pressure and oil film temperature during the whole meshing process of the gear. Luo Pei[12] carried out theoretical calculations on the minimum oil film thickness in the gear meshing stage. Yang Yunfei[13] pointed out that heavy load conditions will lead to the decrease of gear oil film thickness and the increase of oil film pressure

peak. The dynamic change of oil film will significantly affect the vibration characteristics of heavy-duty gears. Reasonable control of oil film parameters can reduce the dynamic impact of gears. The conclusion once again proves the importance of oil film to gears.

The above research provides a certain basis for the calculation of the EHL oil film thickness of the gear, but it is deficient for the EHL oil film thickness of the VH-CATT cylindrical gear. In order to study the EHL oil film thickness of VH-CATT cylindrical gear, the tooth surface equation and tooth surface contact model are derived by gear meshing principle. The principal curvature of the contact tooth surface is solved by Gauss theorem. Based on the EHL theory, the EHL oil film thickness model is established to analyze the influence of different rotational speeds and torques on the oil film thickness of the tooth surface.

2 FORMING PRINCIPLE AND CONTACT ANALYSIS OF VH-CATT CYLINDRICAL GEAR

2.1 TOOTH SURFACE EQUATION

The VH-CATT cylindrical gear is processed by double-edged milling with a large cutterhead. During the machining process, the cutterhead rotates and cuts the blank. After completing a single tooth groove, the blank rotates to the next machining angle, and the complete VH-CATT cylindrical gear is obtained by this cyclic machining, as shown in Figure 1.

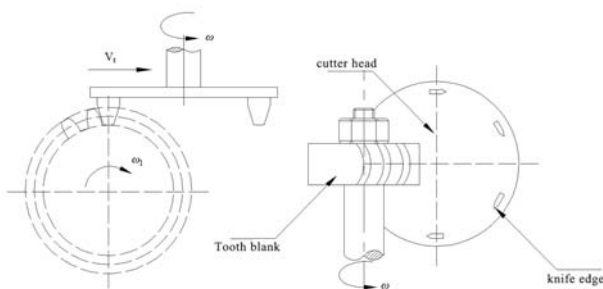


FIG.1 THE SCHEMATIC DIAGRAM SHOWS THE VARIABLE HYPERBOLIC CIRCULAR ARC TOOTH LINE CYLINDRICAL GEAR MACHINED BY DOUBLE-EDGED MILLING OF LARGE CUTTER HEAD

The tooth surface equation of VH-CATT cylindrical gear [2] can be obtained :

$$\begin{cases} x_1 = \left[\left(\mp \frac{\pi m}{4} \mp u_0 \sin \alpha - R_r \right) \cos \theta + (R_r + R_1 \varphi_1) \right] \cos \varphi - (R_1 + u_0 \cos \alpha) \sin \varphi \\ y_1 = \left[\left(\mp \frac{\pi m}{4} \mp u_0 \sin \alpha - R_r \right) \cos \theta + (R_r + R_1 \varphi_1) \right] \sin \varphi + (R_1 + u_0 \cos \alpha) \sin \varphi \\ z_1 = \left(\mp \frac{\pi m}{4} \mp u_0 \sin \alpha - R_r \right) \sin \theta \\ u_0 = \mp \frac{\sin \alpha}{\cos \alpha} \left[\left(R_r \pm \frac{\pi m}{4} \right) \cos \theta - (R_r + R_1 \varphi_1) \right] \end{cases} \quad (1)$$

In the formula : m is expressed as gear modulus, α is the pressure angle, R_T The radius of the cutter head used for processing, R_1 is the radius of the gear dividing circle. φ_1 is the rotation angle of gear blank, and θ is the spread angle of cutter head. Formula 1 distinguishes the concave and convex surface of the gear by positive and negative signs. The convex surface is positive and the concave surface is negative.

2.2 TOOTH CONTACT ANALYSIS OF VH-CATT CYLINDRICAL GEAR

According to the principle of gear combination, TCA analysis of gear pair is carried out. It is necessary to rotate the driving wheel and the driven wheel under the same fixed coordinate system $O_f X_f Y_f Z_f$, so the simplified meshing space coordinate system is established as shown in figure 2.

$O_f X_f Y_f Z_f$ is a fixed coordinate system fixedly connected with the gear rack. The active wheel dynamic coordinate system and the driven wheel dynamic coordinate system are represented by $O_1 X_1 Y_1 Z_1$ and $O_2 X_2 Y_2 Z_2$, respectively. $O_n X_n Y_n Z_n$ and $O_m X_m Y_m Z_m$ are the auxiliary transformation coordinate systems in the process of driven wheel transformation, ϕ_1 is the angle of rotation of the driving wheel around its own Z axis, and ϕ_2 is the angle of rotation of the driven wheel around its own Z axis.

According to the tooth contact analysis in gear geometry[12], during the meshing process of the VH-CATT cylindrical gear pair, the two surfaces of the driving wheel Σ_p and the driven wheel Σ_g must be continuously contacted. The position vector and the normal line overlap at any time, and the TCA mathematical model of the gear in the transmission process can be established as formula 2.

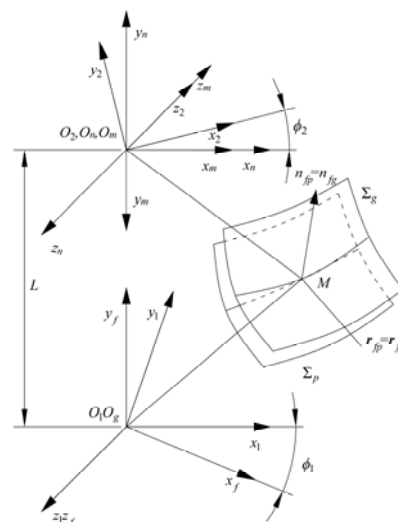


FIG.2 SPACE MESHING COORDINATE SYSTEM

$$\begin{cases} \mathbf{r}_{fp}(\theta_p, \varphi_p, \phi_1) - \mathbf{r}_{fg}(\theta_g, \varphi_g, \phi_2) = \mathbf{0} \\ \mathbf{n}_{fp}(\theta_p, \varphi_p, \phi_1) - \mathbf{n}_{fg}(\theta_g, \varphi_g, \phi_2) = \mathbf{0} \end{cases} \quad (2)$$

In the formula: the rotation angle ϕ_1 of the driving wheel is taken as the knowledge quantity, and the geometric contact model can be solved by substituting the parameters of the gear pair.

2.3 ANALYSIS OF CONTACT TOOTH SURFACE CURVATURE OF VH-CATT CYLINDRICAL GEAR

Curvature is an important parameter affecting the oil film thickness of VH-CATT cylindrical gears. It can reflect the bending shape, contact form, lubrication film forming ability, contact stress and other characteristics of the tooth surface of VH-CATT cylindrical gears. It is the core geometric index to judge the lubrication state and operation safety of gear transmission.

The tooth surface equation of VH-CATT cylindrical gear is rewritten as vector 3, and the unit normal vector is rewritten as Eq. 4 :

$$\mathbf{r}(\theta, \varphi) = x(\theta, \varphi)\mathbf{i} + y(\theta, \varphi)\mathbf{j} + z(\theta, \varphi)\mathbf{k} \quad (3)$$

$$\mathbf{n} = \frac{\mathbf{r}_\theta \times \mathbf{r}_\varphi}{|\mathbf{r}_\theta \times \mathbf{r}_\varphi|} \quad (4)$$

According to Reference[14], the expressions of Gauss curvature K and mean curvature H of the tooth surfaces of the driving wheel Σ_p and the driven wheel Σ_g are :

$$K = \frac{LN - M^2}{EG - F^2} \quad (5)$$

$$H = \frac{LG - 2MF + NE}{2(EG - F^2)} \quad (6)$$

In the formula : $E = \mathbf{r}_\theta^2$ 、 $F = \mathbf{r}_\theta \cdot \mathbf{r}_\varphi$ 、 $G = \mathbf{r}_\varphi^2$ 、 $L = \mathbf{r}_{\theta\theta} \cdot \mathbf{n}$ 、 $M = \mathbf{r}_{\theta\varphi} \cdot \mathbf{n}$ 、 $N = \mathbf{r}_{\varphi\varphi} \cdot \mathbf{n}$.

The principal curvature of the two contact tooth surfaces Σ_p and Σ_g of the VH-CATT cylindrical gear pair in the gear tooth line direction and the tooth profile direction is expressed by k_1 and k_2 , which should satisfy the following quadratic equation :

$$x^2 - Hx + K = 0 \quad (7)$$

Its root is the principal curvature k_1 , k_2 , and the expression is :

$$\begin{cases} k_1 = H + \sqrt{H^2 - K} \\ k_2 = H - \sqrt{H^2 - K} \end{cases} \quad (8)$$

3 LUBRICATING OIL FILM THICKNESS OF VH-CATT CYLINDRICAL GEAR

3.1 ANALYSIS OF CONTACT TOOTH SURFACE VELOCITY OF VH-CATT CYLINDRICAL GEAR

In order to calculate the EHL oil film thickness of the VH-CATT cylindrical gear, the velocity vector at the contact point should be solved first. The specific gear pair design parameters are shown in Table 1.

In order to analyze the relative motion relationship of the gear pair, a simplified coordinate diagram of the motion relationship is established as shown in Fig. 3.

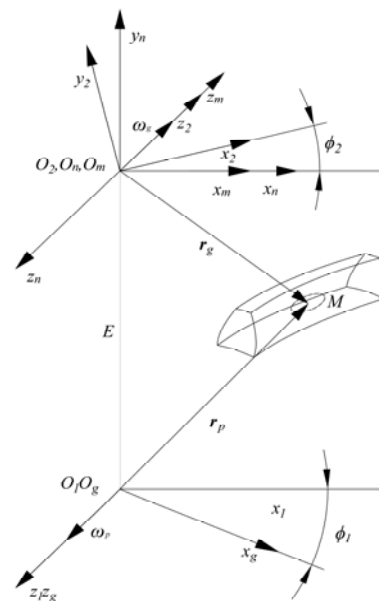


FIG. 3 COORDINATES OF MOTION RELATIONSHIP OF VH-CATT CYLINDRICAL GEAR PAIR

$O_1 X_1 Y_1 Z_1$ and $O_2 X_2 Y_2 Z_2$ represent the active wheel moving coordinate system and the driven wheel moving coordinate system, respectively. $O_f X_f Y_f Z_f$ is a fixed coordinate system fixed to the gear frame, ω_p is the angular velocity when the driving wheel rotates, and ω_g is the angular velocity when the driven wheel rotates. E is the center distance of the VH-CATT cylindrical gear pair, and M is the common contact point of the driving wheel Σ_p and the driven wheel Σ_g .

TABLE 1 DESIGN PARAMETERS

| Design parameter | Gear pair 1 |
|------------------------------------|-------------|
| Modulus m (mm) | 4 |
| Number of driving gear teeth z_p | 21 |

| | |
|--------------------------------------|-----|
| Number of driven gear teeth z_g | 29 |
| Pressure angle α ($^\circ$) | 20 |
| Tooth width B(mm) | 50 |
| Cutter radial RT(mm) | 200 |

In the picture : $\overline{O_1M} = r_p, \overline{O_2M} = r_g, \overline{O_1O_2} = E$.

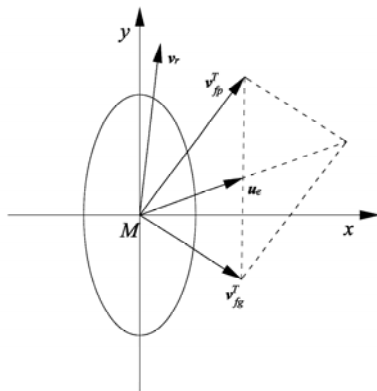


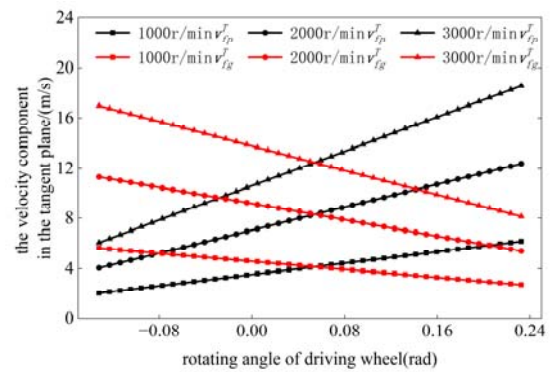
FIG.4 VELOCITY RELATION DIAGRAM

According to Figure 4, we can see the vector relationship between the sliding speed v_r and the entrainment speed u_e and the driving wheel speed component v_{fp}^T and the driven wheel speed component v_{fg}^T [15], so the expression of the relative sliding speed v_r and the entrainment speed u_e is as follows :

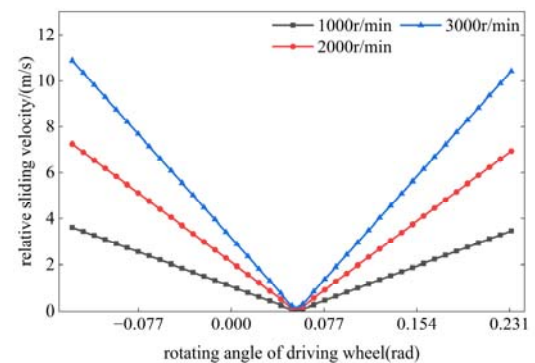
$$v_r = v_{fp}^T - v_{fg}^T \quad (9)$$

$$u_e = \frac{v_{fp}^T + v_{fg}^T}{2} \quad (10)$$

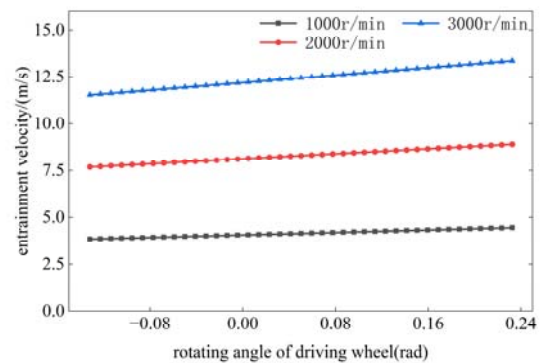
The parameters in the VH-CATT cylindrical gear pair 1 in Table 1 are substituted into the formula of this chapter to solve the velocity vector in one meshing period of the gear pair. At the same time, the input speed ω_p of the driving wheel is set to be 1000r/min, 2000r/min and 3000r/min respectively. The results are as follows :



A) VELOCITY COMPONENT IN TANGENT PLANE



B) RELATIVE SLIDING VELOCITY



C) ENTRAINMENT VELOCITY

FIG. 5 VELOCITY ANALYSIS OF VH-CATT CYLINDRICAL GEAR PAIR 1

It can be seen from the figure that the parameters such as velocity component, relative sliding velocity and entrainment velocity in the tangent plane have obvious rules, and increase with the increase of the input speed ω_p of the driving wheel, and there is an obvious multiple relationship. There is no sudden change in the data, which indicates that the gear transmission is stable during the meshing process.

3.2 EHL OIL FILM THICKNESS MODEL

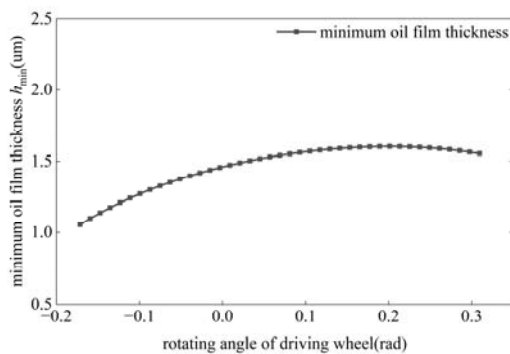
In the actual work of VH-CATT cylindrical gears, due to the influence of errors such as manufacturing, installation, and material deformation, the contact form of gears will change from theoretical line contact to elliptical surface contact, which is the key parameter affecting the thickness of gear oil film. Based on the EHL minimum oil film thickness formula and the central oil film thickness formula [10], the influence of different input speeds and input loads on the oil film thickness of VH-CATT cylindrical gears is expanded.

$$\begin{cases} H_{\min} = 3.63U^{0.68}G^{0.49}W^{-0.073} (1 - e^{-0.68k}) \\ H_{\text{cen}} = 2.69U^{0.67}G^{0.53}W^{-0.067} (1 - 0.61e^{-0.73k}) \end{cases} \quad (11)$$

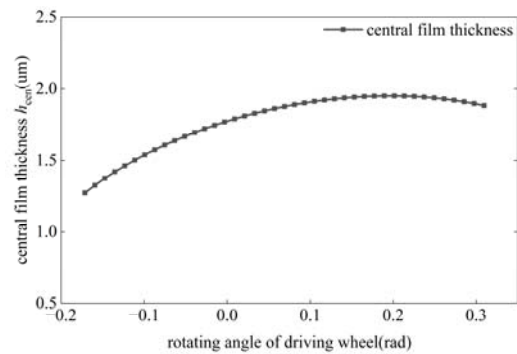
In the formula : H_{\min} is the minimum oil film thickness ; H_{cen} is the central oil film thickness ; U is the velocity parameter ; G is the material parameter ; W is the load parameter ; e is a natural constant ; k is the elliptic parameter.

4 EHL OIL FILM THICKNESS ANALYSIS OF VH-CATT CYLINDRICAL GEAR

Combined with the EHL oil film thickness model, the influence of different input speed and input torque on the minimum and central oil film thickness is analyzed. The parameters of VH-CATT cylindrical gear pair 1 in table 1 are substituted into formula 12, the input speed ωp is 1000r/min, and the input torque T_g is 1000N·m. The minimum EHL oil film thickness and the change trend of EHL center oil film thickness of the gear in a meshing period are obtained as shown in figure 6.



A) MINIMUM OIL FILM THICKNESS



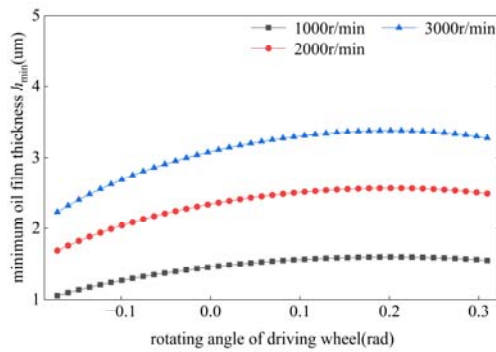
B) CENTRAL OIL FILM THICKNESS

FIG. 6 VH-CATT CYLINDRICAL GEAR HAS THE SMALLEST EHL AND THE CHANGE OF CENTRAL OIL FILM THICKNESS IN A MESHING PERIOD.

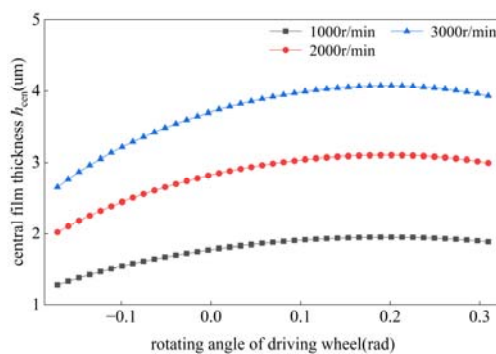
The specific results are shown in Fig.6 a). The minimum oil film thickness of EHL in one meshing period is 1.05 μm , which is located at the meshing position of the driving wheel, and the maximum value is 1.60 μm , which is located at the later stage of the whole meshing process. In Fig.6, b) the minimum value of the central oil film thickness is 1.27 μm at the driving wheel engagement, and the maximum value is 1.95 μm , which is the same as the minimum oil film thickness. In the late stage of the whole meshing process, the rotation angle of the driving wheel in a meshing cycle is $-0.17 \text{ rad} < \phi_1 < 0.31 \text{ rad}$ at 0.20 rad. It can be clearly seen that the minimum oil film thickness has the same trend as the central oil film thickness, showing a trend of increasing first and then decreasing, which is related to the entrainment speed. In the velocity vector analysis diagram, it can be seen that the entrainment velocity increases steadily with the increase of the rotation angle of the driving wheel, which is consistent with the change law of the oil film thickness. When the driving wheel concave surface and the driven wheel convex surface are gradually away from each other, it gradually becomes smaller.

4.1 EFFECT OF ROTATIONAL SPEED ON OIL FILM THICKNESS

In order to analyze the influence of input speed ωp on the forming of VH-CATT cylindrical gear oil film, this chapter analyzes the influence of different input speed ωp on the oil film thickness of VH-CATT cylindrical gear in detail. The input speed ωp is 1000r/min, 2000/min and 3000r/min respectively, and the input torque T_g is 1000N·m. The parameters in VH-CATT cylindrical gear pair 1 in table 1 are substituted into the calculation to obtain figure 7.



A) MINIMUM OIL FILM THICKNESS



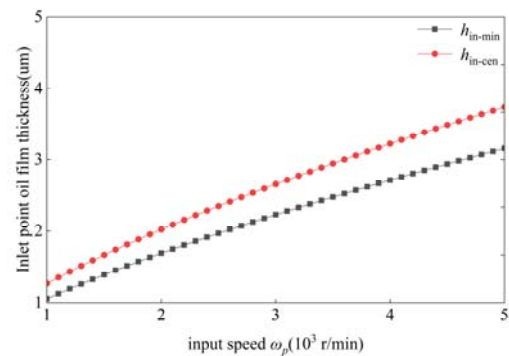
B) CENTRAL OIL FILM THICKNESS

FIG.7 EFFECT OF DIFFERENT INPUT SPEEDS ON OIL FILM THICKNESS

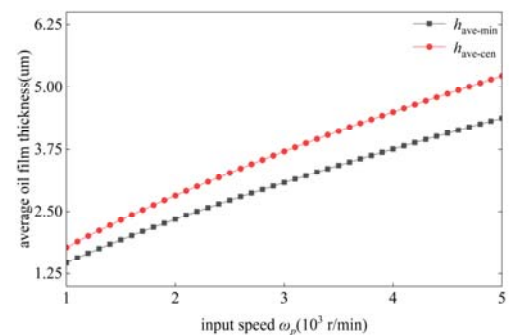
In the case of different input speed ω_p , the change trend of the minimum oil film thickness and the central oil film is consistent with the above, and it can be obviously found that with the increase of the input speed ω_p , it has a positive effect on the formation of the oil film, so that the thickness of the oil film increases, and the thickness of the central oil film is always greater than the minimum oil film thickness. The minimum values of the minimum oil film thickness at different speeds are $1.05\mu\text{m}$, $1.69\mu\text{m}$, and $2.22\mu\text{m}$, respectively. When the gear has just entered the meshing, the maximum values of the minimum oil film thickness are $1.60\mu\text{m}$, $2.56\mu\text{m}$, and $3.38\mu\text{m}$, respectively. At the later stage of gear meshing, about 0.20 rad of the active wheel angle $-0.17\text{ rad} < \phi < 0.31\text{ rad}$ in a meshing cycle, which is the same as the above conclusion. The minimum value of the central oil film thickness is $1.27\mu\text{m}$, $2.02\mu\text{m}$, $2.66\mu\text{m}$. When the gear enters the meshing, the maximum value of the central oil film is $1.95\mu\text{m}$, $3.10\mu\text{m}$, $4.07\mu\text{m}$, which is the same as the minimum oil film thickness in the later stage of gear meshing.

The oil film is the weakest when the gear just enters the meshing, which is the forming place of the smallest oil film in a meshing period. In order to study the change trend of the minimum oil film thickness h_{in-min} and the central oil film thickness h_{in-cen}

at the meshing point, the variable speed ω_p is increased from 1000r/min to 5000r/min by taking the rated input torque T_g as $1000\text{N}\cdot\text{m}$. In order to observe the overall condition of the oil film in a meshing cycle of the VH-CATT cylindrical gear, the average minimum oil film thickness $h_{ave-min}$ and the average central oil film thickness $h_{ave-cen}$ are obtained by summing and averaging the data of each group obtained at different speeds. The results are shown in Fig.8.



A) INPUT SPEED MESHING POINT OIL FILM THICKNESS



B) AVERAGE OIL FILM THICKNESS AT INPUT SPEED

FIG.8 EFFECTS OF DIFFERENT INPUT SPEEDS ON THE OIL FILM THICKNESS AT THE MESHING POINT

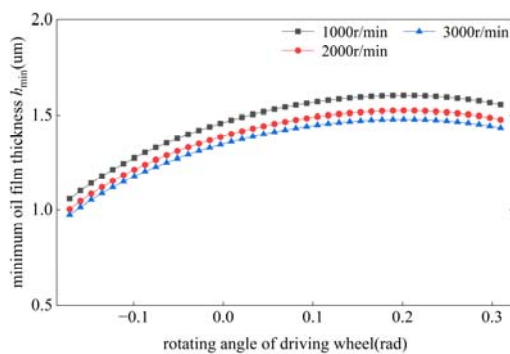
With the increase of speed, h_{in-min} and h_{in-cen} have obvious positive growth. The increase of input speed can obviously improve the oil film forming thickness of the meshing point and reduce the wear condition of the gear when it just enters the meshing state. The $h_{ave-min}$ and $h_{ave-cen}$ are also improved with the input speed. In summary, it can be concluded that the VH-CATT cylinder can better form oil film and reduce tooth surface wear under high speed conditions.

4.2 THE INFLUENCE OF TORQUE ON OIL FILM THICKNESS

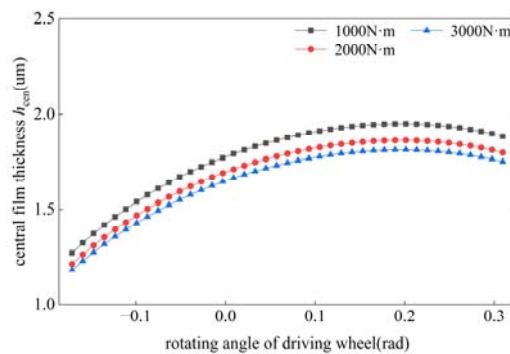
In order to analyze the influence of input torque on the oil film forming of VH-CATT cylindrical gear, this chapter analyzes the influence of different input torque on the oil film thickness of

VH-CATT cylindrical gear in detail. The input torque T_g is taken as $1000\text{N}\cdot\text{m}$, $2000\text{N}\cdot\text{m}$ and $3000\text{N}\cdot\text{m}$ respectively, and the input speed ω_p is taken as $1000/\text{min}$. The parameters in Table 1 VH-CATT cylindrical gear pair 1 are substituted into the calculation as shown in Fig.9.

Under different torques, the forming ability of the gear oil film decreases obviously, and the forming point of the minimum oil film is still at the point of just entering the meshing, and the distribution law is the same as that of the upper section. However, due to the change of input torque, the force on the tooth surface becomes larger, and the deformation of the tooth surface makes the forming ability of the oil film decrease, and the minimum oil film thickness and the central oil film thickness decrease obviously.



A) MINIMUM OIL FILM THICKNESS

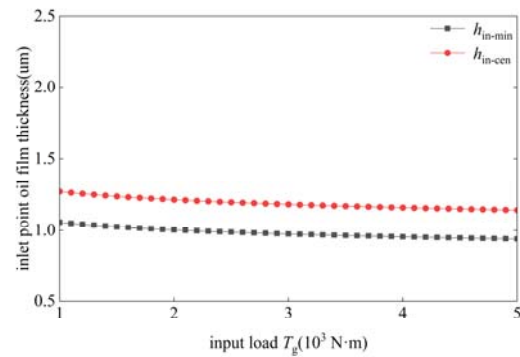


B) CENTRAL OIL FILM THICKNESS

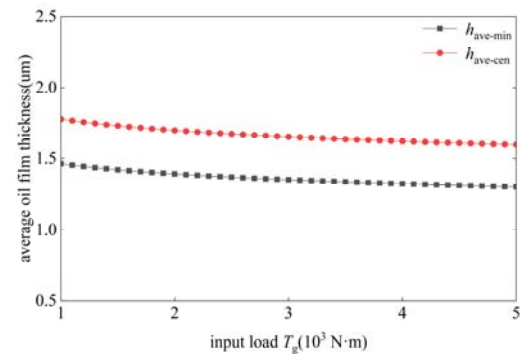
FIG.9 EFFECTS OF DIFFERENT INPUT TORQUES ON OIL FILM THICKNESS

In order to better study the weakest part of the oil film (the meshing point) and the overall change in a meshing period under different input torques, the input speed ω_p is $1000\text{r}/\text{min}$, and the initial value of the torque T_g is set to $1000\text{N}\cdot\text{m}$. Increase to $5000\text{N}\cdot\text{m}$ to observe the minimum oil film thickness h_{in-min} and the central oil film thickness h_{in-cen} of the meshing point. The average minimum oil film thickness h_{in-min} and the

average central oil film thickness h_{in-cen} are obtained by summing the data of each group obtained under different input torques. The results are shown in Fig.10.



A) INPUT TORQUE MESHING POINT OIL FILM THICKNESS



B) AVERAGE OIL FILM THICKNESS OF INPUT TORQUE

FIG.10 EFFECT OF DIFFERENT INPUT TORQUES ON THE OIL FILM THICKNESS AT THE MESHING POINT

Different from the influence of the input speed, the change of the input torque leads to a slight decrease in the oil film forming ability of the VH-CATT cylindrical gear. At the weak area of the oil film forming, the change of the input torque has little effect on it, indicating that the change of the input torque will not greatly affect the forming ability of the oil film at the point of engagement. It can be seen from b) in Fig.10 that the increase of input torque will lead to the decrease of oil film forming ability, but the increase of input torque is greater than the decrease of average oil film. Therefore, VH-CATT cylindrical gear also has good oil film forming ability under high and heavy load conditions.

5CONCLUSION

Based on VH-CAT cylindrical gear, this paper combines TCA analysis and gear space meshing principle to construct TCA mathematical model, and solves the position coordinates of



meshing points. According to the gear geometry, the main curvature and velocity parameters of the contact tooth surface are obtained. Combined with EHL theory, the EHL oil film thickness model of VH-CATT cylindrical gear is established to solve the minimum oil film thickness and central oil film thickness in a meshing period. The results show that :

(1) In the process of continuous transmission, the speed of the gear changes smoothly without mutation, and the overall change of the oil film thickness is continuous and gentle.

(2) The gear oil film is the weakest at the meshing point, and it gradually increases with the rotation of the driving wheel and the increase of the entrainment speed. The central oil film thickness is always greater than the minimum oil film thickness, and the conclusion is the same as the theory.

(3) By changing the input speed and input torque, the data analysis shows that the increase of speed can significantly improve the oil film thickness, and the increase of load only slightly reduces the oil film thickness.

ABOUT THE AUTHOR

Tang Song (2004-), male, from Zunyi, Guizhou Province, undergraduate, the main research areas for mechanical transmission, gear lubrication characteristics, digital design.

REFERENCES

- [1]Di Yutao.Study on the theory of spiral cylindrical gear transmission [D].Harbin : Harbin Institute of Technology, 2006.
- [2]Yan Lili, Hou Li, You Yunxia.Research on the forming principle and maximum contact stress of circular arc gear [J].Mechanical transmission, 2014,38 (08): 26-29+38.
- [3]Ma Dengqiu, Hou Li, Wei Yongqiao, et al. Sliding friction contact mechanics model of circular arc gear based on fractal theory [J]. Chinese Journal of Mechanical Engineering, 2016, 52 (15): 121-127.
- [4]Wei Yongqiao, Yang Haijiang, Guo Rui, et al. Research on power loss of cylindrical gears with variable hyperboloid circular tooth profile [J]. china heavy equipment, 2024, (04): 74-83.
- [5]Ma D, Liu Y, Ye Z, et al. Meshing contact impact properties of circular arc tooth trace cylindrical gear based on rotating knife dish milling process[J]. Mathematical Problems in Engineering, 2021, 2021.
- [6]Zhang Jin.Study on the wear characteristics of variable hyperbolic circular arc cylindrical gears under different contact conditions [D].Lanzhou: Lanzhou University of Technology, 2024.
- [7]REYNOLDS O. On the Theory of Lubrication and Its Application to Mr. Beauchamp tower's experiments, including an experimental determination of the viscosity of olive oil[J]. Philosophical Transactions of the Royal Society of London, 1886, 177: 157-234.
- [8]Blok H . Fundamental mechanical aspects of thin- film lubrication[J]. Annals of the New York Academy of Sciences, 1951, 53: 779-804.
- [9]GRUBIN A N. Fundamentals of the hydrodynamic theory of lubrication of heavily loaded cylindrical surfaces[R]. Moscow: Central Scientific Research Institute for Technology and Mechanical Engineering, 1949: 115-166.
- [10]DOWSON D, HIGGINSON G R. New roller-bearing lubrication formula[J]. Engineering, 1961, 192(4972): 158-159.
- [11]Zhou, Hou, Wei, et al., Establishment and analysis of isothermal elastohydrodynamic lubrication model for circular arc gear [J].Mechanical Transmission, 2015,39 (03): 18-22.
- [12]Research on lubrication characteristics of variable hyperbolic circular arc gear transmission [D].Chengdu: Sichuan University, 2022.
- [13]Yang Yunfei. Study on dynamic characteristics of heavy-duty gears based on elastohydrodynamic lubrication [D]. Xi 'an: Chang 'an University, 2024.
- [14]Litvin, F. L. Gear geometry and applied theory [M]. Shanghai Science and Technology Press, 2008.
- [15]Research on meshing behavior and mixed lubrication characteristics of cylindrical gear with variable hyperbolic arc tooth trace [D].Lanzhou University of Technology, 2022.